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Confining strings in representations with common n-ality

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Abstract: We study the spectrum of confining strings in SU(3) pure gauge theory, by means of lattice Monte Carlo simulations, using torelon operators in different representations of the gauge group. Our results provide direct evidence that the string spectrum is according to predictions based on n-ality. Torelon correlations in the rank-2 symmetric channel appear to be well reproduced by a two-exponential picture, in which the lowest state is given by the fundamental string $\sigma_1 = \sigma$, the heavier string state is such that the ratio σ_2/σ_1 is approximately given by the Casimir ratio $C_{\rm sym}/C_{\rm f} = 5/2$, and the torelon has a much smaller overlap with the lighter fundamental string state.

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The spectrum of confining strings in 4-d SU(N) gauge theories has been much investigated recently. Several numerical studies in the context of a lattice formulation of the theory have appeared in the literature, providing results for color sources associated with representations higher than the fundamental, see e.g. refs. [1]–[6]. General arguments show that the string tension must depend only on the n-ality, $k = \mathrm{mod}(l, N)$, of a representation built out of the (anti-)symmetrized tensor product of l copies of the fundamental representation. The confining string with n-ality k is usually called k-string, and σ_k is the corresponding string tension. Using charge conjugation, $\sigma_k = \sigma_{N-k}$. As a consequence, SU(3) has only one independent string tension determining the large distance behavior of the potential for $k \neq 0$. One must consider larger values of N to look for distinct k-strings. Lattice results for $N = 4, 5, 6$ [6, 5] show a nontrivial spectrum for the k-strings. In particular the data of ref. [6], obtained using color sources in the antisymmetric representations of rank k, turn out to be well reproduced by the sine formula

$$\frac{\sigma_k}{\sigma} \approx \frac{\sin(k\pi/N)}{\sin(\pi/N)}\,, \tag{1}$$

($\sigma \equiv \sigma_1$) within their errors. The sine formula has been suggested by several theoretical works, especially in the context of supersymmetric theories, see e.g. refs. [7, 8] and references therein.

On the other hand, numerical results for different representations with the same n-ality apparently contradict the picture that n-ality is what really matters. For example, in the SU(3) case, Monte Carlo data for the Wilson loops for several representations [2, 3] show apparently area laws up to rather large distances, approximately 1 fm, also for representations with zero n-ality, and the extracted string tensions turn out to be consistent with the so-called Casimir scaling [1]. In the lattice study of ref. [5, 6], considering larger values of N, the k-string tensions were extracted from the torelon masses, i.e. from the exponential decay of correlations of characters of Polyakov lines. In ref. [6], while the antisymmetric representations provided rather clean measurements of σ_k reproducing the sine formula, the numerical results for the symmetric representations suggested different values of the corresponding string tensions. For example, in the case of rank 2, $\sigma_{\mathrm{sym}}/\sigma \gtrsim 2$, which is approximately the value suggested by Casimir scaling or by the propagation of two noninteracting fundamental strings. These results that apparently contradict n-ality have been recently discussed in ref. [9]. They have been explained by arguing that standard color sources, such as Wilson loops and Polyakov lines, associated with representations different from the antisymmetric ones have very small overlap with the stable k-string states, being suppressed by powers of $1/N^2$ in the large-N limit, and in some cases also exponentially. Since $N = 3$ is supposed to be already large, these arguments may explain why the predictions of n-ality have not been directly observed in the numerical simulations, which are limited in accuracy. Moreover, the situation worsens in the case of larger N.

Motivated by this recent work, we decided to return on this issue in the context of the 4-d SU(3) gauge theory. Performing Monte Carlo simulations of the SU(3) gauge theory in its Wilson lattice formulation, we measure correlators of Polyakov lines in the

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representations of rank 1 (fundamental) and 2 of the SU(3) group, in order to check whether their large-distance behaviors, and therefore the values of the corresponding string tensions, are consistent with n-ality.

In our numerical study we use a method based on torelon correlators [10]. The string tensions are extracted from the large-time behavior of "wall-wall" correlators of Polyakov loops in spatial directions, closed through periodic boundary conditions (see e.g. refs. [10, 5]):

$$G_r(t) = \sum_{x_1,x_2} \langle \chi_r[P(0,0;0)]\ \chi_r[P(x_1,x_2;t)] \rangle\,, \tag{2}$$

where $P(x_1,x_2;t) = \Pi_{x_3} U_3(x_1,x_2,x_3;t)$. $U(\mathbf{x};t)$ are the usual link variables, and χ_r is the character of the representation r: $\chi_{\mathrm{f}}[P] = \mathrm{Tr}\,P$ for the fundamental representation, while the two representations of rank 2 (antisymmetric and symmetric, both with n-ality $k=2$) have $\chi_{\mathrm{asym}}[P] = \frac{1}{2}\left((\mathrm{Tr}\,P)^2 - \mathrm{Tr}\,P^2\right)$, $\chi_{\mathrm{sym}}[P] = \frac{1}{2}\left((\mathrm{Tr}\,P)^2 + \mathrm{Tr}\,P^2\right)$, respectively. Note that, since for SU(3) $\chi_{\mathrm{asym}}[U] = \chi_{\mathrm{f}}[U]$, the correlators in the $k=1$ fundamental and $k=2$ antisymmetric representations are identical. We have also studied the adjoint representation (n-ality $k=0$), for which: $\chi_{\mathrm{adj}}[P] = |\,\mathrm{Tr}\,P|^2 - 1$. In this case, one should consider the connected correlator, since $\langle \chi_{\mathrm{adj}}[P] \rangle \neq 0$.

The correlators (2) decay exponentially as $\exp(-m_k t)$, where m_k is the mass of the lightest state in the corresponding representation. Actually, on a finite lattice with periodic boundary conditions we have $G_r(t) \propto \cosh(t - T/2)$, whete T is the temporal size. For a k-loop of size L, the k-string tension is obtained using the relation [10]

$$m_k = \sigma_k L - \frac{\pi}{3L}\,. \tag{3}$$

The last term in eq. (3) is conjectured to be a universal correction, and it is related to the universal critical behavior of the flux excitations described by a free bosonic string [11].

We present results obtained at $\beta = 5.9$ and for two asymmetric lattices $12^3 \times 24$ and $16^3 \times 24$, allowing us to compare the results using Polyakov lines with different length, i.e. $L = 12, 16$. Using our data for the fundamental string tension, see below, and the standard value $\sqrt{\sigma} = 440\,\mathrm{MeV}$, $L = 12, 16$ correspond to approximately 1.5 and 2 fm, respectively. In our simulations we upgraded the SU(3) variables by alternating microcanonical over-relaxation and heat bath steps, typically in a 4:1 ratio. More details on the algorithm can be found in ref. [6]. We collected rather high statistics, ≈ 16M sweeps (considering a sweep as the upgrading of all links of the lattice independently of the algorithm) for $L = 12$ and ≈ 7M sweeps for $L = 16$. Measurements were taken every 20 sweeps. In order to improve the efficiency of the measurements we used smearing and blocking procedures (see e.g. refs. [12]) to construct new operators with a better overlap with the lightest string state. We constructed new super-links using three smearing, and a few blocking steps, according to the value of L, i.e. two for $L = 12$ and four for $L = 16$. These super-links were used to compute improved Polyakov lines. Our implementation of smearing and blocking is as follows [6]: Smearing replaces every spatial link on the lattice according to:

$$U_k(x) \mapsto \mathcal{P}\left\{ U_k(x) + \alpha_s \sum_{\pm(j\neq k)} U_j(x) U_k(x+\hat{\jmath}) U_j^\dagger(x+\hat{k}) \right\}, \tag{4}$$

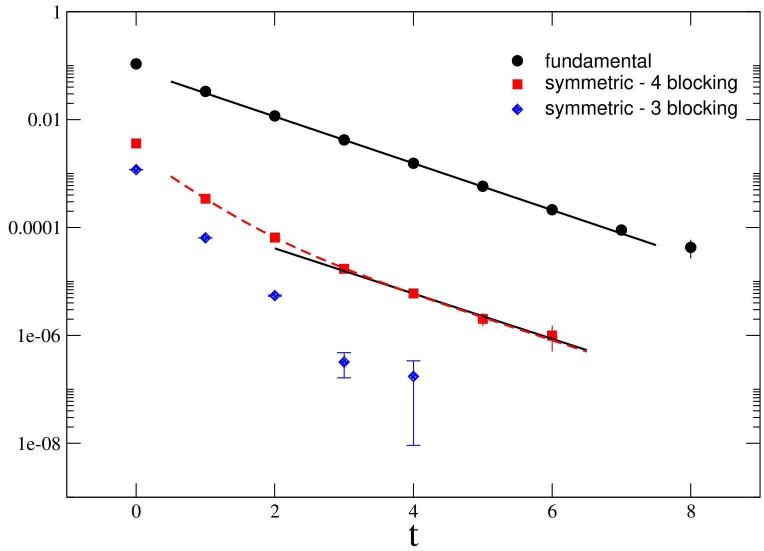

Figure 1: Correlator data for the fundamental (circles) and rank-2 symmetric (squares) representations, with $L = 16$, as a function of the temporal distance t. The top solid line is a hyperbolic cosine fit to data with $t \geq 3$, leading to an estimate of m_1 ; the bottom solid line is a hyperbolic cosine with the *same* exponent, which is well supported by the symmetric representation data for $t \geq 3$. The dashed line represents a fit using the two-exponent Ansatz (6). Data from the rank-2 symmetric representation at one less blocking step is shown in diamonds.

where $\mathcal{P}$ indicates the projection onto SU(N) and the sum only runs on spatial directions. Similarly, blocking replaces each spatial link with a super-link of length $2a$:

$$U_k(x) \mapsto \mathcal{P}\left\{ U_k(x)U_k(x+\hat{k}) + \alpha_f \sum_{\pm(j\neq k)} U_j(x)U_k(x+\hat{j})U_k(x+\hat{j}+\hat{k}))U_j^{\dagger}(x+2\hat{k}) \right\}. \tag{5}$$

The blocking procedure can then be iterated n times to produce super-links of length $2^n a$. The coefficients α_s and α_f can be adjusted to optimise the efficiency of the procedure. We constructed new super-links using $\alpha_s = \alpha_f = 0.5$.

In figures 1 and 2 we show the wall-wall correlators as a function of the distance t in the cases of fundamental and symmetric representations, from the runs with $L = 16$ and $L = 12$ respectively. The data for the correlator in the fundamental representation allow us to accurately determine the fundamental string tension, and the two lattices provide consistent results using eq. (3), i.e. $\sigma = 0.0664(5)$ and $\sigma = 0.0668(3)$ respectively from the $L = 16$ and $L = 12$ runs (obtained by fitting results starting from distances $t = 3, 4$, respectively). On the other hand, such an agreement is not observed in the case of the symmetric representation. However, the $L = 16$ data for the symmetric correlator shows a clear evidence that its asymptotic behavior is controlled by the fundamental string; indeed, fitting the data for $t \geq 3$ we obtain $\sigma_{\rm sym} = 0.070(4)$, in agreement with n-ality. Although data at small t, $t < 3$, show a clear contamination by heavier states, in the symmetric representation case the overlap with the fundamental string state of the source operator, obtained by performing four blocking steps after smearing, appears to be sufficient to show

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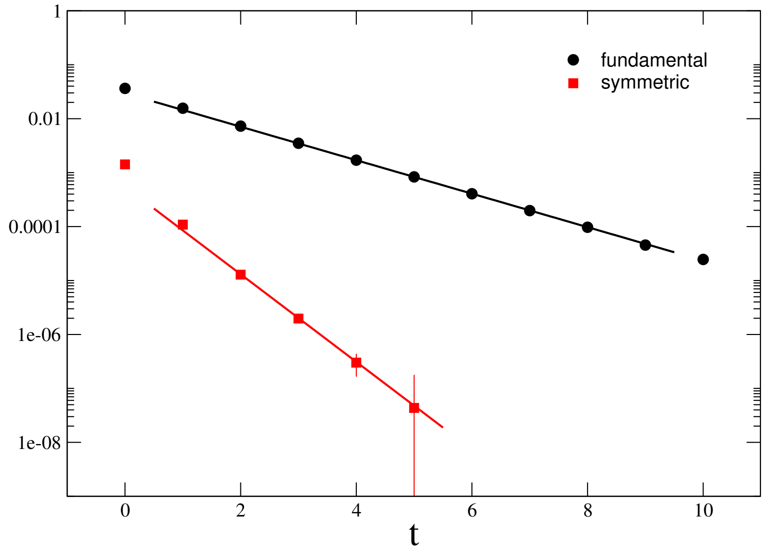

Figure 2: Correlator data for the fundamental (circles) and symmetric (squares) representations, with $L = 12$, as a function of the temporal distance t. Solid lines are exponential fits to data with $t \geq 3$, leading to markedly different values of m_k .

the actual asymptotic behavior before the signal disappears within the error. This is not observed in the $L = 16$ data using the source operator with three blocking steps (one less) and in the $L = 12$ data (where two blocking steps are employed). Up to the distances that we can observe before the signals die off into the noise, the correlators appear to be dominated by the propagation of a much heavier state, which would suggest $\sigma_{\rm sym} \approx 0.16$, whose corresponding ratio $\sigma_{\rm sym}/\sigma \approx 2.4$ is rather close to the Casimir ratio of the two representations, i.e. 5/2.

A simple interpretation of the behaviour of torelon correlations is provided by a picture based on two propagating states; indeed, the numerical results suggest:

$$G_{\rm sym}(t) \simeq c_1 e^{-m_1 t} + c_2 e^{-m_2 t}\,, \tag{6}$$

where $m_i = \sigma_i L - \pi/(3L)$. Due to n-ality, $\sigma_1 = \sigma$. The string tension of the first excited string state is σ_2, and the overlap with the states satisfy $c_1 \ll c_2$. The data on the smaller lattice $L = 12$ already suggests that the ratio σ_2/σ should be approximately equal to the Casimir ratio. As shown in figure 1, the Ansatz (6) fits well the $L = 16$ data at four blocking steps for $t \geq 1$, with $\sigma_2/\sigma \simeq 2.2$ and $c_1/c_2 \simeq 0.12$. Assuming a mild dependence of c_i on L, and given the smallness of c_1, the signal from the fundamental string should only be visible for large enough t:

$$t \simeq \frac{\ln(c_2/c_1)}{(\sigma_2 - \sigma_1)\, L} \tag{7}$$

Clearly, for an acceptable signal-to-noise ratio, t must not grow excessively, and the above equation requires a sufficiently large spatial size L ; this is consistent with the results from the lattice sizes we use.

According to the large-N arguments of ref. [9], the ratio c_1/c_2 should be suppressed at least by a power of $1/N^2$. This would make the observation of the asymptotic $k = 2$ string state, using sources in the symmetric representation, much harder for larger N, thereby explaining the results for $N = 4, 6$ of ref. [6].

In conclusion, the results that we have presented provide direct evidence that the spectrum of confining strings is according to predictions based on n-ality. Torelon correlations in the rank-2 symmetric channel appear to be well reproduced by a two-exponential picture, in which the lowest state is given by the fundamental string $\sigma_1 = \sigma$, the heavier string state is such that the ratio σ_2/σ_1 is approximately given by the Casimir ratio $C_{\rm sym}/C_{\rm f} = 5/2$, and the torelon has a much smaller overlap with the lighter fundamental string state.

In the case of the adjoint representation, since its n-ality is zero, the corresponding mass of the exponential decay in the connected correlator should not depend on L, but it should be related to the propagation of gluelumps. Actually, since the gluelumps have a limited physical size, by increasing L we expect a smaller and smaller overlap of the adjoint Polyakov lines with the lowest states; this makes the evidence for the so-called adjoint string breaking[1] very difficult when using the method applied here, requiring a prohibitive amount of statistics. Indeed, both the $L = 16$ and $L = 12$ data seem to correspond to the propagation of a string state with $\sigma_{\rm adj} \approx 0.145$, and therefore $\sigma_{\rm adj}/\sigma \approx 2.1$, which is again rather close to the corresponding Casimir ratio $9/4$.

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